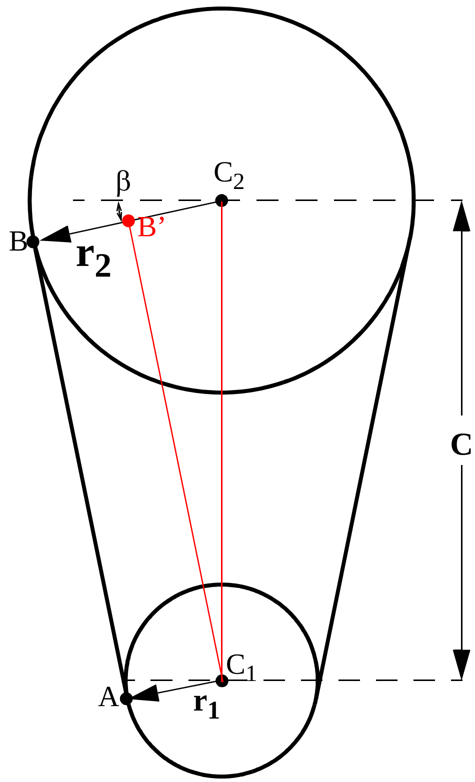
Given two pulleys with sizes r_1 and r_2, with centers C_1 and C_2 separated by a distance C as drawn below, the problem is to calculate the belt length B as a function of r_1 , r_2 and C. Also calculate the partial circumference of the pulleys touched by the belt.

I have drawn the radius arrows to point to the points of tangency. I have named the points of tangency A and B, and the centers C_1 and C_2 respectively.

Angles C_2BA and C_1AB are both right angles because A and B are points of tangency. Because rays C_1A and C_2B are both perpendicular to the same line, they are parallel. That means that the angle b formed by radius r_2 and a horizontal line is the same angle as radius r_1 forms with a horizontal line.



Next consider another line segment C_1B' formed exactly parallel to line segment AB, but translated such that one end is at point C_1. This line is drawn in red. Further consider a line C_1C_2, also depicted in red.

Angle $B'C_1C_2$ is also angle b.

Line segment $B'C_2$ has length r_2 - r_1.

Line segment C_1C_2 has length C.

Line segment $B'C_1$ has length: $\overline{B'C_1} = \sqrt{C^2 - (r_2 - r_1)^2}$

Consider the right triangle $B'C_1C_2$: $\sin(\beta) = \frac{r_2 - r_1}{C}$

Now consider the fraction of the circumference of the larger wheel that is touched by the belt. The included angle is $180 + 2\beta$. Similarly, the angle of tangency on the lower wheel is $180 - 2\beta$. The length of the belt consists of the sum of the partial circumference around the upper wheel, twice the length of line segment AB, and the partial circumference around the lower wheel.

The partial circumference of the upper wheel has length: $2\pi r_2 \times \frac{180 + 2\beta}{360}$

The partial circumference of the lower wheel has length: $2\pi r_1 \times \frac{180 - 2\beta}{360}$

Let:
$$dR = r_2 - r_1$$
$$\overline{AB} = \sqrt{C^2 - (r_2 - r_1)^2}$$

Then belt length B is expressed:

$$B = 2\overline{AB} + \pi(r_2 + r_1) + \frac{\pi dR}{90} \times \operatorname{asin}\left(\frac{dR}{C}\right)$$